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Analysis of photon statistics with Silicon Photomultiplier

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ABSTRACT: The Silicon Photomultiplier (SiPM) is a novel silicon-based photodetector, which represents the modern perspective of low photon flux detection. The aim of this paper is to provide an introduction on the statistical analysis methods needed to understand and estimate in quantitative way the correct features and description of the response of the SiPM to a coherent source of light.

KEYWORDS: Photon detectors for UV, visible and IR photons (solid-state) (PIN diodes, APDs, Si-PMTs, G-APDs, CCDs, EBCCDs, EMCCDs etc); Analysis and statistical methods

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1 Introduction

The Silicon Photomultiplier (SiPM) is a novel silicon-based photodetector, which represents the modern perspective of low photon flux detection [1, 2]. It consists of an array of space-distributed micro sensors. Each micro sensor is as small as $20 \times 20 \mu\text{m}^2$ with the present technologies and is able to detect down to the single quantum of light. The array is detecting the incoming photon flux.

The SiPM shows an excellent performance, including high detection efficiency (about 25%–60%) for the visible range of light, fast timing response (about 30 ps), operation condition at room temperature with bias 20–60 V, not sensitivity to external magnetic fields. This opened the way of its successful application in high energy physics, nuclear medicine, homeland security and many other areas, where the SiPM replaced the traditional Photomultiplier Tube [3].

The SiPM is developed and compatible with the standard CMOS technology [4]. The new possibilities of its design and performances are deeply connected with the success of this technology in the recent years. In particular the integration of SiPM sensitive elements and electronics on the same chip and the reduction of the microcell area without deteriorating the photon detection efficiency are now possible within the 3D CMOS technology option [5]. This novel advanced SiPM design is called Fully Digital Silicon Photomultiplier [6, 7].

The ability of single photon detection down to the small microcell area and with fast timing response makes the novel SiPM sensitive to the detection and analysis of photon fluctuations and statistics at unprecedented level. For this reason the SiPM is also called in literature Quantum Photo Detector [6]. It has interesting implications and possible applications in the recent developing field of Quantum Optics.

However the study of the photon statistics and of photon correlations requires a detailed analysis of the experimental data obtained from the SiPM at a high level of trust and precision. The few

attempts in this direction in the literature assume only a strong statistical modelling of certain characteristics of the SiPM, as photodetection efficiency, dark rate and cross-talk, without considering the quality of the measurement and of the detector itself [8].

The aim of this paper is to provide an introduction on the statistical analysis methods needed to understand and estimate in quantitative way the correct features and description of the response of the SiPM to a coherent source of light. The analysis is a result of a series of exercises proposed at the Second International Summer School on Intelligent Signal Processing for Frontier Research and Industry held in Paris, July 14–24, 2014 [9].

The paper is organized as follows. In section 2 we outline the physical principles of the SiPM photodetection and we define the task and challenges of the statistical analysis of the SiPM signal on the physical and operational basis; in section 3 we describe the response of the SiPM to a coherent light source and we describe the statistical analysis procedure on the basis of the outlined physical and operational principles.

2 Physical principles of the SiPM operation

2.1 Physical principles

The process of detection of the single photon is determined by the nature of the photon, the physics of the interaction of photons with matter and the mechanism of electric pulse formation, i.e. the conversion of the photon energy into the electric signal, which is finally analyzed in the measurement system.

Visible light photons have almost the minimal possible energy value in nature. The energy of a 500 nm wavelength photon (green) is estimated as low as 2.2 eV. The detection of such small energy value is the fundamental problem of single photon detection. The area of Quantum Optics analyses the detection of the single photon from a photon distribution, involving the study of the intrinsic quantum mechanics of the electromagnetic field [10].

The photoelectric effect is the main physics process of photon interaction with the matter for the visible range of light. It provides the conversion of one photon into one electron-hole pair. The electronic system needs to measure such small amount of electric charge.

Silicon semiconductor detection structures are formed of two regions with different conductivity (p- and n-doped silicon). By applying a reverse bias to such structure, a depleted area is formed with low concentration of carriers and in-built electric field. The structure is designed specifically to match the photon detection condition via photoelectric effect within the depleted area.

The process of creation of an electron-hole pair due to the photoelectric effect is shown schematically on figure 1. Photons with energy higher than the band-gap of the semiconductor materials are absorbed in the depleted area creating an electron-hole pair. The two carriers are separated by the electric field and drift respectively to the positive enhanced n-region and negative enhanced p-region. The charge carriers are collected on the electrodes and pass through the external circuit, which transports the signal to the measurement system. However the charge of a single electron is as low as 10^{-19} C and its registration is a very complicated task, because of the noise level of readout electronics. It is estimated that modern electronics have sensitivity for signal above about 300 electrons, or approximately 10^{-17} C. It means that the minimum number of electron-holes pairs, to which the readout electronics and measurement system are sensitive, is 300.

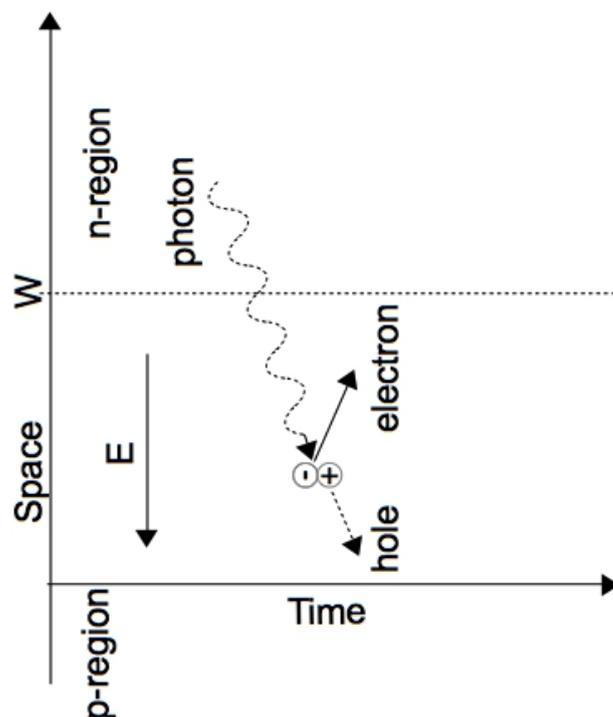


Figure 1. Principle of photon detection via photoelectric effect in a Silicon Detection Structure.

Modern detection structures overcome this problem with intrinsic amplification up to 10^5 – 10^6 . This is the basic idea of single photon detection. The avalanche process via impact-ionization is the physical mechanism responsible of the intrinsic multiplication in modern semiconductor detection structures. In high electric field, usually higher than 10^5 V/cm, free carriers are accelerated and could reach the energy higher than the ionization energy of valence electrons [11]. The consequence of the secondary impact ionization is the avalanche multiplication of electron-hole pairs and the increase of the value of the electric field corresponding to the initial charge created by the interaction of the photons.

The impact ionization parameters of electrons and holes are used to characterize the dynamics of the avalanche process. They are defined as the inverse mean free path of the two carriers. According to the strength of the electric field, two different avalanche configuration can occur.

Two types of avalanche process in silicon detection structures shown on the figure 2. At electric field lower than about 10^4 V/m (figure 2, left) the impact ionization of holes is much lower than the impact ionization of electrons. As a result the avalanche process is generated only by carriers of the electron type and is self-quenched, when the electron carriers reach the border of the depleted region. This is the typical configuration and working principle of the Avalanche Photodiodes (APD). It is important to note that in this case the multiplication factor is strongly determined by the thickness of the depletion region and the quenching mechanism is affected by the statistical fluctuation of the photoelectric effect interaction point and of the number of carriers produced in the avalanche. This statistical fluctuation is mainly responsible of the degradation of the conventional APD signal in response to the low photon flux.

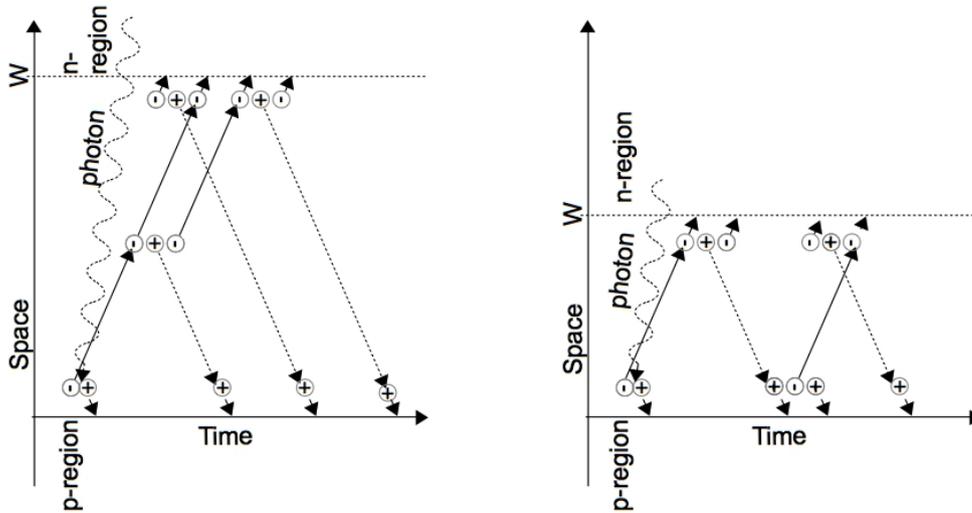


Figure 2. Two types of avalanche process in silicon detection structures. (Left) avalanche process carried by electron only; (Right) avalanche process carried by both electrons and holes.

In contrast to APD, the SiPM is operated at electric field higher than 10^5 V/m (figure 2, right). At this high field strength the impact ionizations of electrons and holes are equivalent and both carriers contribute to the avalanche. It is important to underline some key-features of this condition. First the avalanche is self-sustaining and the gain is not limited by the depletion region thickness; the depletion region could be as thin as the mean free path of the carriers in order to get infinite multiplication. Second carriers or photon impact point statistical fluctuations do not affect the multiplication factor; the breakdown condition has infinite multiplication and it is quenched by external active or passive elements, not by the fluctuation of the carriers as in the APD case. Third the statistical fluctuation of the produced signal is determined only by the properties of the quenching element.

An additional physical mechanism happening during the avalanche process is the emission of optical photons due to direct inter-band, intra-band transitions and Bremsstrahlung.

It is important to be noted that in such silicon detection structures thermally generated electron-hole pairs can initiate the avalanche process also. Their contribution is undistinguishable from the electron-hole pairs generated by photoelectric effect.

On this physics basis, the task and challenge of the statistical analysis of the Silicon Photomultiplier response to the low photon flux can be defined as the sensitivity to the intrinsic features of the avalanche breakdown process and to their impact in the SiPM signal, in order to discriminate them from external systematics introduced by the measurement system, which should not contribute significantly respect to the small expected fluctuations of the SiPM output signal.

2.2 Principle of operation of the SiPM

The operational principle of the Silicon Photomultiplier is based on the use of quenched avalanche breakdown process in the silicon micro sensors (microcells) with an implemented quenching element. The schematic of such photon detecting microcell is shown in figure 3 (left) One possible solution is a serial resistor to the microcell. After the initiation of the avalanche breakdown process

the current through the structure raises and causes a voltage drop on the resistor with consequent voltage drop on the bias applied to the pn-junction of the sensitive micro-cell. The avalanche stops, when the voltage drop on the quenching resistor lowers the voltage applied to the pn junction below the breakdown voltage. After the structure is quenched, a recovery time is necessary to allow any free or stored charge to be swept from the active region of the device and to restore the excess bias across the microcell.

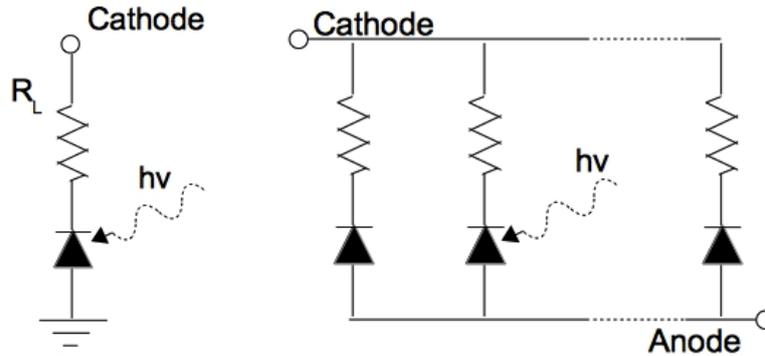


Figure 3. Structure of the single sensitive microcell (left) and schematics of the array of microcells composing the SiPM (right)

The SiPM is composed of a space-distributed array of such microcells (figure 3, right). The output of the SiPM is the analogue sum of the signal of all microcells, connected in parallel to a common electrode structure.

Two aspects are important to be mentioned. First the output of each microcell is identical in response to the single photon. Second fluctuations in response to single photon are due to the inter-pixel uniformity of microcells, pn junctions and quenching resistors. The modern technology allows controlling the manufacturing process of microelectronics devices in physical dimensions down to 130 nm and less. Moreover controlled performances in concentration profiles and advanced high resistive polysilicon technology allow satisfying the uniformity requirement for a correct operation of the SiPM in the detection of low photon flux.

On this operational basis, the task of statistical analysis of the Silicon Photomultiplier response to the low photon flux can be defined as the sensitivity to the intrinsic technological properties of the SiPM manufacturing and their discrimination from the external measurement system fluctuations, which do not have to contribute to the overall signal fluctuations more than the intrinsic technological performances of the SiPM.

3 The statistics of photon detection in SiPM

3.1 Experimental setup

We build an experimental set-up consisting of a coherent light source coupled with the sensitive area of a 1 mm^2 SiPM. Both SiPM and light source are contained in a optically isolated environment, in order to minimize the impact of external light sources. The light source is driven by a high frequency pulser which provides also a trigger to the experiment. The SiPM output is amplified

before integration, for compatibility between the SiPM output and the measurement system. It has to be noted that the internal amplification of the SiPM is enough for the signal discrimination above the electronic noise and the additional external amplification is only required to match the specifications of the measurement system. A Charge to Digital Converter (QDC) performs the SiPM pulse integration, digitalizes the information and communicates with an analysis computing system through a standard VME BUS communication system.

The statistical analysis software is installed on the analysis computing system and allows for online and offline data analysis. The computing exercises proposed in the INFIERI schools were based on the statistical analysis software applied to an already collected dataset. The analysis performed in order to characterize the SiPM on the basis of the above-mentioned physical and operational principles of the SiPM and on the basis of the physical properties of the light. The next subsections provide the detailed description and solution of the exercises proposed at the school.

3.2 The single photo electron spectrum — definition and mathematical model

A typical spectrum of the response of the Silicon Photomultiplier to the low photon flux of a coherent light source is shown in figure 4.

The histogram shows the number of registered events with a specific charge. The charge is expressed in equivalent number of electrons ($1 e^- = 1.60210^{-19} C$). The width of each histogram bin corresponds to 3.97×10^6 electrons.

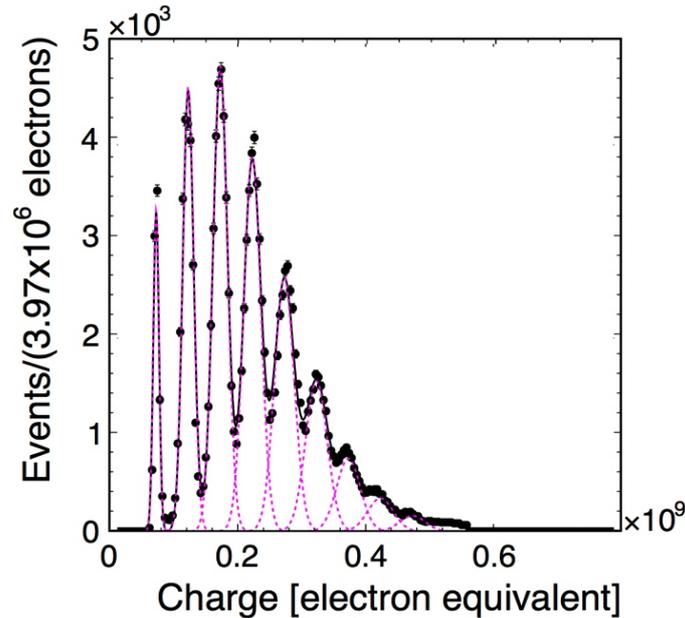


Figure 4. Response of the SiPM to a light source.

A peak structure is visible in the spectrum. Being the output of the SiPM the analogue sum of the microcells, the first one corresponds to 0 detected photons, the second one to 1 detected photon, the n^{th} one to n detected photons.

The shape of each peak is well approximated with a Gaussian distribution, with average position μ_n and variance σ_n . The most general distribution function describing the SiPM spectrum in response to coherent light is hence the sum of n gaussians G :

$$f(x) = \sum_{i=0}^{n-1} \alpha_i G(x, \mu_i, \sigma_i) + \left(1 - \sum_{i=0}^{n-1} \alpha_i\right) G(x, \mu_n, \sigma_n) \quad (1)$$

where the variable x is the charge and the weighting coefficients α_n are related by the normalization condition.

The task of the statistical analysis of the SiPM response to the coherent light source is hence the fit of the SiPM Single photon electron spectrum with the fitting function (1) and the extraction and analysis of the parameters μ_n σ_n α_n for each peak.

Although simple in shape, the statistical model consisting of a composition of distribution functions includes a non-trivial statistical operation. Not only each Gaussian $G(x)$ should follow in fact the normalization property of a distribution function, but also the sum function $f(x)$ should be normalized to 1. It follows that each fraction α_n is defined in the range between $[0, 1]$, but under the strong correlation normalization condition:

$$\sum_{i=0}^n \alpha_i = 1 \quad (2)$$

The correlation of these parameters makes the fit handling and precision challenging. The statistical analysis framework ROOFIT [12] provides a solid basis for the integration of such mathematical model into the needs of statistical analysis. ROOFIT is a statistical analysis framework broadly used for the data analysis in High Energy Physics experiments and consists of a large collection of statistical and numerical methods.

Important for the analysis of the SiPM spectrum, the normalization condition of the composed distribution function is obtained with numerical integration, allowing a stable fitting procedure. The function $f(x)$ is fitted to the experimental data using a Maximum Likelihood method approach. The normalization condition is recalculated dynamically with numerical integration at each step of the minimization of the Likelihood function.

The results shown in the following sections are obtained implementing the fitting function and the data analysis within the ROOFIT statistical framework.

3.3 Statistical analysis on the basis of physical and operational principles of the SiPM

As stated in section 2, the aim of the statistical analysis on the basis of the physical and operational principles of the SiPM is the study of the fluctuations of the SiPM signal corresponding to the detection of one or more photons and the determination of physical and technological effects. In this respect it is important to quantify the fluctuation sources which depend on the experimental measurement system and have some impact on the overall fluctuation of the SiPM signal.

The first peak (peak-0) of the single photoelectron spectrum corresponds to the condition of no detected photons. Its width hence does not depend on the properties of the SiPM and is only determined by the noise of the measurement system. This first peak is commonly defined as *pedestal* and its width is σ_0 .

The second peak (peak-1) of the single photoelectron spectrum corresponds to the condition of one detected photon, or of one firing microcell. As the array of microcells is readout in parallel and there is no information about the particular firing microcell. The first peak collects hence the integral information of the properties of the response of each independent microcell composing the SiPM. Its width σ_1 expresses the fluctuations originating from the not-uniformity of the microcells, not uniformity of the quenching resistor affecting hence the not uniformity of the gain. We define the intrinsic contribution of the technological and physical parameters of the single microcells in the array composing the SiPM as σ_{int} . In addition the fluctuations of the peak-1 signal include the electronics noise of the experimental setup described by σ_0 . Being the intrinsic microcell fluctuations and the electronic noise uncorrelated, the width of the peak-1 is expressed as:

$$\sigma_1^2 = \sigma_0^2 + \sigma_{\text{int}}^2 \quad (3)$$

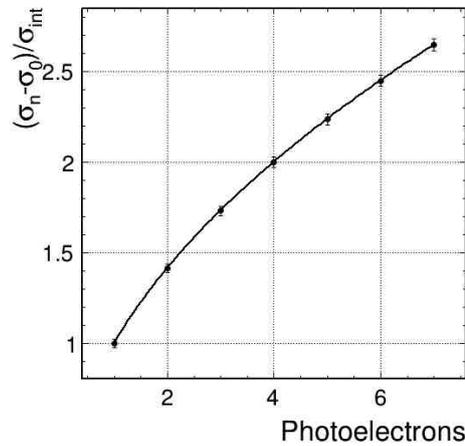


Figure 5. Dependence of relative width on number of photoelectrons

The third peak (peak-2) of the single photoelectron spectrum corresponds to the condition of two detected photons, or of two firing microcells. Following the reasoning outlined for peak-1, it is possible to conclude that the fluctuations of the signal in peak-2 correspond to the intrinsic fluctuation of any two firing microcells randomly chosen from the array and the electronic noise. Extending this concept to the n^{th} peak it is possible to obtain the expression:

$$\sigma_n^2 = \sigma_0^2 + n\sigma_{\text{int}}^2 \quad (4)$$

The position of the peaks contains also the information about the avalanche process. The position of the first peak (peak-0) is determined only by the average electronic noise from the experimental setup registered in the integration gate and is expressed by the fitting parameter μ_0 . The position of the second peak (peak-1) is indicated as μ_1 and represents the average number of electron-holes produced in the avalanche breakdown process in one microcell added to the electronic noise. Extending this reasoning to the further peaks we obtain the following relation:

$$\mu_n = \mu_0 + n \times g \quad (5)$$

where g indicates the gain of the SiPM, i.e. the number of electron-hole pairs produced by one microcell in one avalanche breakdown process. The distance between the peaks is uniform and corresponds to the number of electrons produced in the avalanche in one microcell.

The function (1) is fitted to the SiPM spectrum in figure 4. The result of the fit is shown on figure 4 as the black continuous line. Each Gaussian component is shown as the blue dotted line. The variance of the first peak (peak-0) corresponding to the fluctuations of the experimental setup is $\sigma_0 = (4.39 \pm 0.04)10^4$ electrons. The value of the intrinsic fluctuation of the single microcell response is extracted from the analysis of the fitted σ_n using the relation (4) as $\sigma_{\text{int}} = (7.30 \pm 0.02)10^4$ electrons.

The dependence of the fitted variance of the n^{th} peak on the number of firing microcells n is shown in figure 5, where the continuous line shows the correct functionality of square root of the number of microcells expected from equation (4). This result shows that the assumption of independence of the microcells is correct.

The gain of the SiPM is obtained analyzing the fitted μ_n according to the relation (5) and is estimated as $g = (4.974 \pm 0.002)10^5$ electrons.

According to the physical and operational principles of the SiPM introduced in section 2, in this specific example one detected photon in a microcell generates one electron-hole pair via photoelectric effect. The avalanche breakdown process is initiated and the quenching mechanism through the quenching resistor determines the gain of $(4.974 \pm 0.002)10^5$ electrons. However the array of microcells present a slight not uniformity. Not precise manufacturing of the quenching resistor or of the pn junctions contribute to the gain with a microcell-by-microcell fluctuation with a variance of $(7.30 \pm 0.02)10^4$ electrons. The relevance of the cell-by-cell fluctuation on the average cell gain determines the possibility of the single photon detection in the SiPM. In other words the SiPM single photon resolution is defined as:

$$R_1 = \frac{\sigma_{\text{int}}}{g} \quad (6)$$

In this example the single photon resolution is estimated as $(14.67 \pm 0.02) \%$.

The single photon resolution does not depend on the statistical fluctuations of the number of electron-hole pairs produced in the avalanche. For each microcells this number has a very limited fluctuation and is only determined by the external quenching element. The single photon resolution depends on the not uniformity of the microcells in the array forming the SiPM. As the outputs of the microcells are summed in parallel the not-uniformity of the microcells reflects into a gaussian fluctuation of the output signal. The measurement and precise estimation of the single photon resolution gives hence an hint on the technology reliability.

It has to be noted that this analysis is possible only if the contribution of the electronic noise introduced by the experimental measurement setup is negligible respect to the intrinsic fluctuations of the detector. In this specific example we obtain a ratio between intrinsic and noise fluctuations $\sigma_{\text{int}}/\sigma_0$ approximately equal to 1.7, which is high enough to resolve with accuracy the intrinsic SiPM single photon resolution.

SiPM single photoelectron spectra with experimental measurement fluctuations significantly higher than intrinsic fluctuations should be considered not trustable for this kind of analysis and in general they do not provide interesting information about extraction of physical and technological

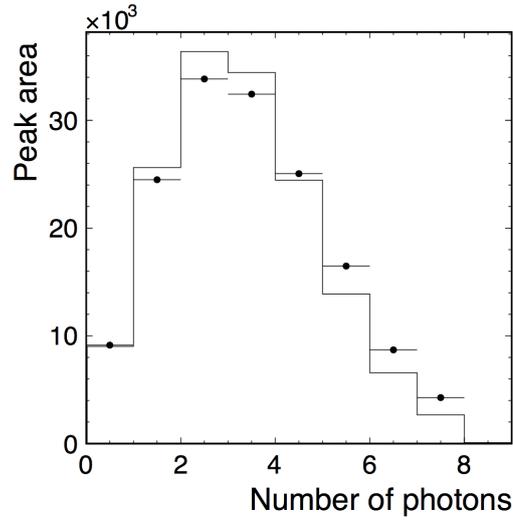


Figure 6. Distribution of the number of events under each peak (black points) and expected Poisson distribution (continuous light)

quantities. In case the analysis of the single photon resolution is limited by the experimental measurement noise, more effort should be done in a more accurate measurement, before attempting a precise analysis of the experimental data.

3.4 Statistical analysis on the basis of physical properties of the light

One of the most interesting features of a coherent light source is the Poisson distribution of the number of emitted photons [10]. The unique SiPM single photon resolution described in the previous section allows to sample the photon statistics from a coherent light source with unprecedented resolution.

The finite photon detection efficiency of the SiPM does not affect this measurement. The only effect is that the Poisson light statistics is sampled with a binomial distribution representing the probability of photon detection. It is however a known result in statistics that the convolution of a binomial and Poisson distribution generates a Poisson distribution.

In the case of a positive result of the statistical analysis described in the previous section the fitting function (1) can be simplified reducing the number of free parameters as:

$$f(x) = \sum_{i=0}^{n-1} \alpha_i G\left(x, \mu_0 + ig, \sqrt{\sigma_0^2 + i\sigma_{\text{int}}^2}\right) + \left(1 - \sum_{i=0}^{n-1} \alpha_i\right) G\left(x, \mu_0 + ng, \sqrt{\sigma_0^2 + n\sigma_{\text{int}}^2}\right) \quad (7)$$

where we used the relations (4) and (5), whose validity we demonstrated in the previous section.

It is important to stress again that this substitution is valid only in case of negligible σ_0 respect to σ_{int} , i.e. in the conditions of the statistical analysis outlined in section 3.2.

This statistical analysis focuses on the weighting fit parameters α_i , which were used only as nuisance and normalization parameters in the previous analysis. These parameters correspond to the number of events in each peak. In particular the number of events under the i^{th} peak is estimated

as:

$$N_i = \alpha_i N, i = 0 \dots n - 1; N_n = N - \sum_{i=0}^{n-1} N_i \quad (8)$$

where N is the total statistics collected in the histogram.

The distribution of the fitted number of events corresponding to each peak in the Silicon Photomultiplier spectrum is shown in figure 6 (black points). The mean value of the expected Poisson light distribution can be estimated from the area of the first peak (peak-0). The number of times 0 photons are detected is in fact related to the mean value of the Poisson distribution λ according to the formula:

$$N_0 = N e^{-\lambda} \quad (9)$$

In the example of this paper the mean of the expected Poisson distribution is estimated as $\lambda = 2.84$. The expected Poisson distribution is plot on figure 6 (continuous line). A discrepancy between measured data points and expected Poisson distribution is observable. In particular peaks corresponding to a higher number of photons are over-represented.

The effect can be understood considering the production of thermally generated electron-hole pairs in the microcells and the generation of optical photons during the avalanche process. The former mechanism, called *dark rate*, initiates avalanche within a microcell without the detection of a photon, the signal being indistinguishable from the photon itself. The latter mechanism, called *cross talk*, initiates avalanche in the microcells close to the firing microcell, in which the optical photon was emitted. In both cases the microcell signal is identical to the photon detection from a photon source. These two effect cause the total number of firing microcells to be higher than the effective number of photons detected, explaining the discrepancy between the expected Poisson distribution and the experimental spectrum.

Additional experimental analysis is needed in order to quantify the cross talk and the dark rate and correct the measured number of photons.

4 Conclusions

The application of the Silicon Photomultiplier in the field of Quantum Optics for the study of the intrinsic properties of light has a promising perspective. However the interpretation of the experimental data obtained in the measurement of the response of this photodetector to the light source requires special attention.

In this paper we demonstrated that the statistical extraction of the intrinsic single photon detection resolution of the SiPM is possible only if the contribution of the measurement setup to the signal fluctuation is negligible compared to the intrinsic fluctuations of the SiPM signal. The single photon detection resolution is interpreted as the effect of the interpixel not-uniformity and it is not related to any statistical fluctuation of the avalanche process, being the latter determined only by the external quenching element.

The SiPM is a suitable detector for the sampling of single photon from photon distributions. In this paper we demonstrated that the intrinsic Poisson distribution of a coherent light source is sampled by the SiPM, with the small perturbation of dark rate and cross talk, which could be corrected with further experimental and statistical analysis.

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